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The final fate of the rolling tachyon

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Abstract: We propose an alternative interpretation of the boundary state for the rolling tachyon, which may depict the time evolution of unstable D-branes in string theory. Splitting the string variable in the temporal direction into the classical part, which we may call "time" and the quantum one, we observe the time dependent behaviour of the boundary. Using the fermion representation of the rolling tachyon boundary state, we show that the boundary state correctly describes the time-dependent decay process of the unstable D-brane into a S-brane at the classical level.

Keywords: D-branes, Tachyon Condensation.

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1. Introduction

The time evolution of unstable states has been a perennial research subject in various branches of theoretical physics. The most recent problem which appears on the stage is the fate of unstable D-branes in string theory. Since A. Sen brought up this problem [1], called rolling tachyon, it has been one of the most important research themes in string theory [2 − 30]. It was asserted that the unstable D-brane decays as the open string tachyons condensate [31 − 46] on the D-brane and it may be described by an exact time-dependent classical solution of string theory.

The rolling tachyon can be described in the sigma model approach by introducing an exactly marginal boundary term to the world-sheet string action,

$$S = -\frac{1}{4\pi}\int_M d\tau d\sigma \partial X^0 \cdot \partial X^0 + \oint_{\partial M} d\sigma \left(Ae^{X^0} + Be^{-X^0}\right) + \dots, \tag{1.1}$$

where the second term describes the boundary interaction corresponding to the open string tachyon field with A and B constants. We abbreviate the string action for the spatial string coordinates X^i, $i = 1, \dots, 25$ and use units where $\alpha' = 1$. Note the negative signature of the kinetic term for X^0 which indicates that it is the time coordinate. Choosing $A = g/2$ and $B = 0$, we can study an unstable D-brane which decays into a more stable configuration. The tachyon profile with this simplest choice is called half-S-brane [3]. The quantum theory of this sigma model is well defined if we take a Wick rotation to Euclidean time, $X^0 \to iX^0$. After Wick rotation we get a conformal theory for a non-compact boson with a periodic boundary potential. It is well known that the conformal field theory of this kind can be described in terms of the boundary state, which can be given as an exact sum of Ishibashi states of the SU(2) current algebra. Sen observed that this boundary state can be used to depict the time evolution of the unstable D-branes. Since then, various aspects of the time dependent process driven by the tachyon instability have been explored by numerous authors. However, our understanding the final fate of the unstable D-brane and the exact dynamics of the rolling tachyon is not yet complete. The exact time dependent description is only available for the first few lower levels. If we expand the boundary state in the bosonic oscillator basis, we get

$$|B\rangle = f(t)|0,t\rangle + g(t)\alpha_{-1}\tilde{\alpha}_{-1}|0,t\rangle + \cdots . \tag{1.2}$$

By some algebra we have [1]

$$f(t) = \sum_j (-\pi g e^t)^{2j} = \frac{1}{1+\pi g e^t}, \quad g(t) = f(t) - 2 = \frac{1}{1+\pi g e^t} - 2 \tag{1.3}$$

where the reverse Wick rotation is taken.

As $t \to \infty$, we find

$$f(t) \to 0, \quad g(t) \to -2. \tag{1.4}$$

Since the stress tensor for the tachyon profile is given as

$$T_{00} = -T_p, \quad T_{ij} = \delta_{ij} T_p f(t), \tag{1.5}$$

$T_{ij} \to 0$ while T_{00} is independent of time as $t \to \infty$. It implies that the pressure vanishes in this limit. This observation led Sen [47, 48] to propose that the decay product may be pressureless tachyon matter [49–53].

One may attempt to evaluate the higher level components in the expansion of the boundary state eq. (1.2). Constable and Larsen [25] developed an efficient techniques to evaluate the higher level components using the Polyakov string path integral. They compute large classes of the higher level components explicitly, in particular,

$$B^{(N,N)}(t) = f(t) - \frac{2}{N}\sum_{n=0}^{N-1}(N-n)(-\pi g e^t)^n, \tag{1.6}$$

where $B^{(N,N)}$ is the coefficient of the terms $\alpha_N \tilde{\alpha}_{-N}|0\rangle$ in the expansion of the boundary state eq. (1.2). An interesting point is that all the coefficients, $B^{(N,N)}$, diverge

$$B^{(N,N)} \to -\frac{2}{N}(-\pi g)^{(N-1)} e^{(N-1)t} \qquad \text{as } t \to \infty \tag{1.7}$$

except for $B^{(1,1)}(t) = g(t) \to -2$. However, this divergent behaviour of $B^{(N,N)}(t)$ contrasts with our expectation: The boundary state $|B\rangle$ is supposed to describe the time dependent process of the rolling tachyon at classical level in a well defined manner. It leads us to speculate that the bosonic oscillator basis may not be a suitable basis to describe the time dependent description of the rolling tachyon.

The boundary condition which follows from the world-sheet string action eq. (1.1) is

$$\left.\left(\frac{1}{2\pi}\frac{\partial}{\partial\tau}X(\tau,\sigma) - \frac{g}{2}e^X\right)\right|_{\tau=0} = 0. \tag{1.8}$$

Note that when $g \to -\infty$, the boundary condition reduces to the Neumann condition. If we denote the zero mode part of X by t, the effective coupling becomes ge^t and the boundary condition implies that the unstable object is a D-brane initially in the far past. As $t \to \infty$, the boundary interaction becomes effectively stronger. And it has been conjectured [54] that in the far future the unstable D-brane may become a S-brane [55 − 66], which is a localized object in the temporal direction. The main subject of this paper is to show how one can realize this intuitive picture in the framework of the boundary state formulation, which takes into account the full string degrees of freedom.

2. Time evolution of the boundary state

One of the distinctive features of string theory which differentiates it from the ordinary quantum field theory is that "time" is embedded in the dynamical variable X along the temporal direction as its zero mode. Due to this distinctive feature often time-dependent description become subtle in string theory. In order to get the time-dependent description of a given dynamical process we should carefully factor the time from the dynamical degrees of freedom

Here we take a simple strategy to split the string variable X as follows

$$X = t + \hat{X}. \tag{2.1}$$

Here t corresponds to the classical part of X, which we may call the "time" and $\hat{X}$ denotes collevtively the quantum degrees of freedom. We regard t as a modular parameter of the string state (or the target space) and $\hat{X}$ as the dynamical variable describing quantum fluctuations around it. In order to have a well-define quantum system we may take the Wick rotation for X:

$$X_E = iX = i\hat{X} + it. \tag{2.2}$$

Hereafter we adopt the Wick rotated string variable X_E throughout the paper. (We also drop the subscript 'E', i.e., in the followings X will denote X_E.)

Time evolution of some simple boundary states in one dimension

Recently we developed [67, 68] a free fermion representation of the boundary conformal field theory for the rolling tachyon, generalizing the work of Polchinski and Thorlacius [69] for the open string to the closed string. In this fermion theory the boundary interaction becomes a simple fermion current operator. As a result an explicit, compact, exact expression of the boundary state has been obtained. Since the marginal boundary term becomes a bilinear operator in terms of fermion fields, it would be more appropriate to discuss the time dependent description using the fermion basis rather than the bosonic oscillator basis.

Once we factor the time, we find that the fermion boundary state behaves distinctively depending on the boundary condition it satisfies. Let us consider a simple one dimensional system first where it has only one string coordinate. In two dimensions fermions and bosons are mapped to each other by

$$\psi_L(z) \;=\; e^{-i\frac{\pi}{2}p_R} \;:e^{-i\sqrt{2}X_L(z)}:, \quad \psi^\dagger_L(z) \;=\; e^{i\frac{\pi}{2}p_R} \;:e^{i\sqrt{2}X_L(z)}: \tag{2.3a}$$

$$\psi_R(\bar{z}) \;=\; e^{-i\frac{\pi}{2}p_L} \;:e^{i\sqrt{2}X_R(\bar{z})}:, \quad \psi^\dagger_R(\bar{z}) \;=\; e^{i\frac{\pi}{2}p_L} \;:e^{-i\sqrt{2}X_R(\bar{z})}: \tag{2.3b}$$

where $X_R(\bar{z})$ and $X_L(z)$ are the right- and left-moving boson fields, respectively. The left- and right-moving boson operators are defined by the mode expansions:

$$X_L(\tau+i\sigma) = \frac{1}{\sqrt{2}}x_L - \frac{i}{\sqrt{2}}p_L(\tau+i\sigma) + \frac{i}{\sqrt{2}}\sum_{n\neq 0}\frac{\alpha_n}{n}e^{-n(\tau+i\sigma)}, \tag{2.4a}$$

$$X_R(\tau-i\sigma) = \frac{1}{\sqrt{2}}x_R - \frac{i}{\sqrt{2}}p_R(\tau-i\sigma) + \frac{i}{\sqrt{2}}\sum_{n\neq 0}\frac{\tilde{\alpha}_n}{n}e^{-n(\tau-i\sigma)}. \tag{2.4b}$$

The non-vanishing commutators are

$$[x_L, p_L] = i \quad , \quad [x_R, p_R] = i \tag{2.5a}$$

$$[\alpha_m, \alpha_n] = m\delta_{m+n} \quad , \quad [\tilde{\alpha}_m, \tilde{\alpha}_n] = m\delta_{m+n}. \tag{2.5b}$$

The Neumann and Dirichlet boundary conditions for the bosonic string are given as

$$X_L(0,\sigma)\,|N\rangle = X_R(0,\sigma)\,|N\rangle\,, \tag{2.6a}$$

$$X_L(0,\sigma)\,|D\rangle = -X_R(0,\sigma)\,|D\rangle\,. \tag{2.6b}$$

These boundary conditions are transcribed in the fermion theory as follows

$$\psi_L(0,\sigma)\ \ |N\rangle = i\psi_R^\dagger(0,\sigma)\,|N\rangle\ ,\ \ \psi_L^\dagger(0,\sigma)\,|N\rangle = i\psi_R(0,\sigma)\ \ |N\rangle\,, \tag{2.7a}$$

$$\psi_L(0,\sigma)\ \ |D\rangle = -i\psi_R(0,\sigma)\,|D\rangle\ ,\ \ \psi_L^\dagger(0,\sigma)\,|D\rangle = -i\psi_R^\dagger(0,\sigma)\ \ |D\rangle\,. \tag{2.7b}$$

We may construct the boundary states which satisfy these conditions[1]

$$|N\rangle = :\exp\left\{i\int\frac{d\sigma}{2\pi}\left(\psi^\dagger\tilde{\psi}^\dagger + \psi\tilde{\psi}\right)\right\}:|0\rangle, \tag{2.8a}$$

$$|D\rangle = :\exp\left\{i\int\frac{d\sigma}{2\pi}\left(\tilde{\psi}^\dagger\psi - \psi^\dagger\tilde{\psi}\right)\right\}:|0\rangle. \tag{2.8b}$$

If we introduce the time t explicitly, the Neumann condition does not change but the Dirichlet condition changes into

$$X_E|D;t\rangle = (X_L + X_R)\,|D;t\rangle = it|D;t\rangle. \tag{2.9}$$

The corresponding fermion boundary condition becomes

$$\begin{aligned}\psi_L(0,\sigma)\ \ |D;t\rangle &= -ie^{\sqrt{2}t}\psi_R(0,\sigma)\,|D;t\rangle\ ,\\ \psi_L^\dagger(0,\sigma)\,|D;t\rangle &= -ie^{-\sqrt{2}t}\psi_R^\dagger(0,\sigma)\ \ |D;t\rangle\,.\end{aligned} \tag{2.10}$$

Hence the boundary state $|D;t\rangle$ is constructed to be in the fermion representation as

$$|D;t\rangle =: \exp\left\{i\int\frac{d\sigma}{2\pi}(e^{-\sqrt{2}t}\tilde{\psi}^\dagger\psi - e^{\sqrt{2}t}\psi^\dagger\tilde{\psi})\right\}:|0\rangle. \tag{2.11}$$

Since the Neumann state does not change,

$$|N;t\rangle = |N;0\rangle =: \exp\left\{i\int\frac{d\sigma}{2\pi}\left(\psi^\dagger\tilde{\psi}^\dagger + \psi\tilde{\psi}\right)\right\}:|0\rangle. \tag{2.12}$$

[1]As is well-known, the fermion theory has two secotrs; the NS-NS sector and the R-R sector. Explicit expressions of the fermion boundary states are slightly different in each sector. Here we only discuss the NS-NS sector for the sake of simplicity. See [67] for more detailed expressions.

Time evolution of some simple boundary states in two dimensions

In order to fermionize the boundary conformal field theory for the rolling tachyon we introduce an auxiliary free boson Y as in refs. [67, 68]

$$\phi_{1L} = \frac{X_L + Y_L}{\sqrt{2}} \ , \quad \phi_{1R} = \frac{X_R + Y_R}{\sqrt{2}} \tag{2.13a}$$

$$\phi_{2L} = \frac{X_L - Y_L}{\sqrt{2}} \ , \quad \phi_{2R} = \frac{X_R - Y_R}{\sqrt{2}}. \tag{2.13b}$$

In the two dimensional system, described by two bosons X and Y, we have four simple boundary states $|D,N\rangle$, $|D,D\rangle$, $|N,N\rangle$, $|N,D\rangle$ where the first and second label is the boundary condition for X and Y bosons respectively. These simple boundary states are given in terms of the fermion fields respectively [67] as

$$\begin{aligned}
|N,N\rangle &= :\exp\left\{ i \int \frac{d\sigma}{2\pi} \left(\psi_1 \tilde{\psi}_1 + \psi_1^\dagger \tilde{\psi}_1^\dagger + \psi_2 \tilde{\psi}_2 + \psi_2^\dagger \tilde{\psi}_2^\dagger \right) \right\} : |0\rangle \\
|N,D\rangle &= :\exp\left\{ \int \frac{d\sigma}{2\pi} \left(\psi_1^\dagger \tilde{\psi}_2 - \psi_2^\dagger \tilde{\psi}_1 - \tilde{\psi}_1^\dagger \psi_2 + \tilde{\psi}_2^\dagger \psi_1 \right) \right\} : |0\rangle \\
|D,N\rangle &= :\exp\left\{ \int \frac{d\sigma}{2\pi} \left(\psi_1^\dagger \tilde{\psi}_2^\dagger - \tilde{\psi}_1^\dagger \psi_2^\dagger + \tilde{\psi}_1 \psi_2 - \psi_1 \tilde{\psi}_2 \right) \right\} : |0\rangle \\
|D,D\rangle &= :\exp\left\{ i \int \frac{d\sigma}{2\pi} \left(\psi_1^\dagger \tilde{\psi}_1 - \psi_2^\dagger \tilde{\psi}_2 - \tilde{\psi}_1^\dagger \psi_1 + \tilde{\psi}_2^\dagger \psi_2 \right) \right\} : |0\rangle
\end{aligned} \tag{2.14}$$

Now we introduce t by factoring the zero mode of X explicitly. Since $|N,D\rangle$ and $|N,N\rangle$ are independent of time (the zero mode of X), they do not change in time

$$|N,N;t\rangle = |N,N;0\rangle = |N,N\rangle, \quad |N,D;t\rangle = |N,D;0\rangle = |N,D\rangle. \tag{2.15}$$

The time dependence of the boundary state $|D,N\rangle$ follow from its t dependent boundary condition

$$\begin{aligned}
\psi_{1L}(0,\sigma)|D,N;t\rangle &= e^t \psi_{2R}^\dagger(0,\sigma)|D,N;t\rangle, \\
\psi_{2L}(0,\sigma)|D,N\rangle &= e^{-t} \psi_{1R}^\dagger(0,\sigma)|D,N;t\rangle, \\
\psi_{1L}^\dagger(0,\sigma)|D,N;t\rangle &= -e^{-t} \psi_{2R}(0,\sigma)|D,N\rangle, \\
\psi_{2L}^\dagger(0,\sigma)|D,N\rangle &= -e^{t} \psi_{1R}(0,\sigma)|D,N;t\rangle.
\end{aligned} \tag{2.16}$$

Thus, the boundary state $|D,N;t\rangle$ is constructed as

$$|D,N;t\rangle =: \exp\left\{ \int \frac{d\sigma}{2\pi} \left(e^t \psi_1^\dagger \tilde{\psi}_2^\dagger - e^{-t} \tilde{\psi}_1^\dagger \psi_2^\dagger + e^{-t} \tilde{\psi}_1 \psi_2 - e^{-t} \psi_1 \tilde{\psi}_2 \right) \right\} : |0\rangle. \tag{2.17}$$

One can construct the boundary state $|D,D;t\rangle$ also in a similar way. If t is explicitly factored, the boundary condition for $|D,D;t\rangle$ is read as

$$\begin{aligned}
\psi_{1L}(0,\sigma)|D,D;t\rangle &= ie^t \psi_{1R}(0,\sigma)|D,D;t\rangle, \\
\psi_{2L}(0,\sigma)|D,D;t\rangle &= -ie^{-t} \psi_{2R}(0,\sigma)|D,D;t\rangle,
\end{aligned} \tag{2.18a}$$

$$\begin{aligned}
\psi_{1L}^\dagger(0,\sigma)|D,D;t\rangle &= ie^{-t} \psi_{1R}^\dagger(0,\sigma)|D,D;t\rangle, \\
\psi_{2L}^\dagger(0,\sigma)|D,D;t\rangle &= -ie^{t} \psi_{2R}^\dagger(0,\sigma)|D,D;t\rangle.
\end{aligned} \tag{2.18b}$$

The boundary state $|D,D;t\rangle$ satisfying these boundary conditions is easily obtained as

$$|D,D;t\rangle =: \exp\left\{i\int\frac{d\sigma}{2\pi}\left(e^t\psi_1^\dagger\tilde{\psi}_1 - e^{-t}\psi_2^\dagger\tilde{\psi}_2 - e^{-t}\tilde{\psi}_1^\dagger\psi_1 + e^t\tilde{\psi}_2^\dagger\psi_2\right)\right\} : |0\rangle. \quad (2.19)$$

Time evolution of the rolling tachyon

We may recall the exact boundary state for the rolling tachyon in fermion theory [67]

$$\begin{aligned} |B,D;0\rangle \ = \ : \exp\Bigg\{\int\frac{d\sigma}{2\pi}\Big[&\left(\psi_1^\dagger\tilde{\psi}_2 - \psi_2^\dagger\tilde{\psi}_1 - \tilde{\psi}_1^\dagger\psi_2 + \tilde{\psi}_2^\dagger\psi_1\right) \\ &-g\pi i\left(\psi_1^\dagger\tilde{\psi}_1 - \tilde{\psi}_2^\dagger\psi_2\right)\Big]\Bigg\} : |0\rangle \end{aligned} \quad (2.20)$$

The time dependent boundary state $|B,D;t\rangle$ can be obtained by simply factoring t

$$\begin{aligned} |B,D;t\rangle \ = \ : \exp\Bigg\{\int\frac{d\sigma}{2\pi}\Big[&\left(\psi_1^\dagger\tilde{\psi}_2 - \psi_2^\dagger\tilde{\psi}_1 - \tilde{\psi}_1^\dagger\psi_2 + \tilde{\psi}_2^\dagger\psi_1\right) \\ &-ge^t\pi i\left(\psi_1^\dagger\tilde{\psi}_1 - \tilde{\psi}_2^\dagger\psi_2\right)\Big]\Bigg\} : |0\rangle \end{aligned} \quad (2.21)$$

The time dependent boundary conditions which $|B,D;t\rangle$ satisfies are:

$$\left(\psi_{1L}(0,\sigma) - i\pi g e^t\psi_{2L}(0,\sigma)\right)|B,D\rangle \ = \ \psi_{2R}(0,\sigma)|B,D\rangle, \quad (2.22a)$$

$$-\psi_{2L}(0,\sigma)|B,D\rangle \ = \ \psi_{1R}(0,\sigma)|B,D\rangle, \quad (2.22b)$$

$$\left(\psi_{2L}^\dagger(0,\sigma) + i\pi g e^t\psi_{1L}^\dagger(0,\sigma)\right)|B,D\rangle \ = \ \psi_{1R}^\dagger(0,\sigma)|B,D\rangle, \quad (2.22c)$$

$$-\psi_{1L}^\dagger(0,\sigma)|B,D\rangle \ = \ \psi_{2R}^\dagger(0,\sigma)|B,D\rangle. \quad (2.22d)$$

The boundary state $|B,D;t\rangle$ may be considered as the quantum state of the unstable D-brane probed by the closed string around $\langle X\rangle = t$.

3. The final fate of the rolling tachyon

We may rewrite the boundary state $|B,D;t\rangle$ eq. (2.21) as follows

$$|B,D;t\rangle \ = \ : \exp\left\{i\int\frac{d\sigma}{2\pi}\left[\Psi_L^\dagger M_1\Psi_R - \Psi_R^\dagger M_2\Psi_L\right]\right\} : |0\rangle \quad (3.1)$$

where

$$M_1 = \sigma_2 + \frac{ge^t\pi}{2}(I+\sigma_3) = \begin{pmatrix} ge^t\pi & -i \\ i & 0 \end{pmatrix}, \quad M_2 = \sigma_2 + \frac{ge^t\pi}{2}(I-\sigma_3) = \begin{pmatrix} 0 & -i \\ i & ge^t\pi \end{pmatrix} \quad (3.2)$$

and

$$\Psi_L = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \Psi_R = \begin{pmatrix} \tilde{\psi}_1 \\ \tilde{\psi}_2 \end{pmatrix}.$$

Note that Ψ_L and Ψ_R are the local perturbative basis in string theory and we are free to choose the most suitable one to describe the system. Any two basis are related to each other by a similarity transformation. The analysis of the time dependent behaviour in the limit where $t \to \infty$ would be simpler if we choose the perturbative basis where the matrices M_1 and M_2 are diagonalized. What we should be concerned with is the spectrum of the string state in the limit.

Both matrices M_1 and M_2 are Hermitian and have the same characteristic equation: Their eigenvalues are

$$\lambda_\pm = \frac{ge^t\pi}{2} \pm \sqrt{1+\left(\frac{ge^t\pi}{2}\right)^2}. \tag{3.3}$$

The (normalized) eigenvectors for M_1 with the eigenvalues $\lambda_\pm$ respectively are

$$\frac{1}{\sqrt{\lambda_\pm^2+1}}\begin{pmatrix}\lambda_\pm \\ i\end{pmatrix}. \tag{3.4}$$

Defining

$$S_1 = \begin{pmatrix}\frac{\lambda_+}{\sqrt{\lambda_+^2+1}} & \frac{\lambda_-}{\sqrt{\lambda_-^2+1}} \\ \frac{i}{\sqrt{\lambda_+^2+1}} & \frac{i}{\sqrt{\lambda_-^2+1}}\end{pmatrix}, \tag{3.5}$$

we diagonalize M_1 by a similarity transformation

$$M_1 \to S_1^{-1}M_1S_1 = \begin{pmatrix}\lambda_+ & 0 \\ 0 & \lambda_-\end{pmatrix}. \tag{3.6}$$

Simultaneously we transform the fermion fields $\Psi^\dagger_{-n}$ and $\tilde{\Psi}_{-n}$ with $n>0$

$$\Psi^\dagger_{-n} \to \Psi^\dagger_{-n}S_1, \quad \tilde{\Psi}_{-n} \to S_1^{-1}\tilde{\Psi}_{-n}. \tag{3.7}$$

In order to preserve the canonical anti-commutation relations among fermion fields we must also transform Ψ_n and $\tilde{\Psi}^\dagger_n$ $(n>0)$ as follows

$$\Psi_n \to S_1^{-1}\Psi_n, \quad \tilde{\Psi}^\dagger_n \to \tilde{\Psi}^\dagger_n S_1. \tag{3.8}$$

Following the same steps we diagonalize M_2. If we choose the normalized eigenvectors for M_2 as

$$\frac{1}{\sqrt{\lambda_\mp^2+1}}\begin{pmatrix}-i \\ \lambda_\mp\end{pmatrix}, \tag{3.9}$$

the transformation matrix S_2 is given as

$$S_2 = \begin{pmatrix}-\frac{i}{\sqrt{\lambda_-^2+1}} & -\frac{i}{\sqrt{\lambda_+^2+1}} \\ \frac{\lambda_-}{\sqrt{\lambda_-^2+1}} & \frac{\lambda_+}{\sqrt{\lambda_+^2+1}}\end{pmatrix}. \tag{3.10}$$

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The second term in the exponent of eq. (2.21) can be diagonalized by taking a similarity transformation

$$M_2 \to -S_2^{-1} M_2 S_2, \quad \Psi_{-n} \to i S_2^{-1} \Psi_{-n}, \quad \tilde{\Psi}^\dagger_{-n} \to i \tilde{\Psi}^\dagger_{-n} S_2, \quad n > 0 \tag{3.11}$$

Here an extra phase i is introduced to take care of the unwanted phase we will get after diagonalization. The canonical anti-commutation relation is preserved if we transform $\Psi^\dagger_n$ and $\tilde{\Psi}_n$ accordingly as

$$\Psi^\dagger_n \to -i \Psi^\dagger_n S_2, \quad \tilde{\Psi}_n \to -i S_2^{-1} \tilde{\Psi}_n, \quad n > 0 \tag{3.12}$$

Upon diagonalizing, we have

$$M_1 = \frac{g\pi}{2} I + \sqrt{1 + \left(\frac{g e^t \pi}{2}\right)^2}\, \sigma_3, \tag{3.13a}$$

$$M_2 = -\frac{g\pi}{2} I + \sqrt{1 + \left(\frac{g e^t \pi}{2}\right)^2}\, \sigma_3. \tag{3.13b}$$

(Note also that under this similarity transformation the world sheet string Hamiltonian is invariant.)

In the limit where $t \to \infty$,

$$M_1 \to \begin{pmatrix} e^t & 0 \\ 0 & -e^{-t} \end{pmatrix}, \quad M_2 \to \begin{pmatrix} e^{-t} & 0 \\ 0 & -e^t \end{pmatrix}. \tag{3.14}$$

Here we translate t such that

$$t \to t + t_0, \quad g\pi e^{t_0} = 1. \tag{3.15}$$

After diagonalizing the exponent, in the limit where $t \to \infty$, the boundary state $|B, D; t\rangle$ behaves precisely as $|D, D; t\rangle$ eq. (2.19)

$$|B, D; t\rangle \to |D, D; t\rangle, \quad \text{as} \;\; t \to \infty \tag{3.16}$$

Of course, in the far past where $t \to -\infty$, the boundary state reduces to the boundary state for a D-brane

$$|B, D; t\rangle \to |N, D; t\rangle = |N, D\rangle, \quad \text{as} \;\; t \to -\infty \tag{3.17}$$

Thus, in the far future the unstable Dp-brane can be viewed as a Sp-brane if the local perturbative basis is appropriately chosen.

4. Conclusions

The time dependent decay process of the unstable D-brane is one of the most important subjects in string theory. Splitting the classical part from the quantum part of the string coordinate variable in the temporal direction and identifying it as time, we observe the

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time dependent behaviour of the boundary state, which describes the unstable D-brane. The classical part called time is a non-dynamical part of the zero mode, which may be regarded as a modular parameter of the target space-time. The boundary state proposed by Sen [1] is supposed to be a classical solution of the string theory which should depict the decay process at the classical level. The final fate of the unstable D-brane, which should be described by the proposed boundary state, has been one of focal points of recent studies [2 − 30]. It has been also conjectured that the final decay product may be the S-brane, all of which tangential dimensions are spacelike [54 − 66]. However, how such a classical description of the decay process of the unstable D-brane can be realized in a consistent manner with the rolling tachyon was an open question.

Here in this paper we propose an alternative interpretation of the boundary state for the rolling tachyon to show that the boundary state correctly depicts the decay process of the unstable D-brane into a S-brane at classical level. The strategy we take is to separate the non-dynamical part from the string coordinate variable in the temporal direction and identify it as time and the rest as the quantum degrees of freedom. Then we apply the Wick rotation to quantum part only to have a well-defined quantum system as suggested by Sen and examine the time dependent behaviour of the boundary state. In the far past the boundary state trivially reduces to that for a D-brane. And we observe that in the far future the boundary state approaches that of the S-brane if we choose the local string perturbative basis appropriately. The boundary state delineates continuous transition of the D-brane into a S-brane at classical level. It is worth while to note that the fermion representation of the boundary state is quite useful to find the most suitable perturbative basis. It is expected that the proposed boundary state for the rolling tachyon sources would produce the closed string field components $B^{(N,N)}$, as given in eq. (1.6). It would be an important task to calculate the time dependent closed string field components, using the fermion representation, to elucidate the nature of the decay of the unstable D-brane in the framework proposed in this paper.

The unstable D-brane may undergo further some quantum decay process by emitting closed strings. The quantum decay process of the unstable D-brane has been already studied by numerous authors [3, 70 − 79]. It would be an interesting work to combine the classical process discussed here and the quantum process studied in the previous works to get a consistent unified description of the decay of the unstable D-brane.

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